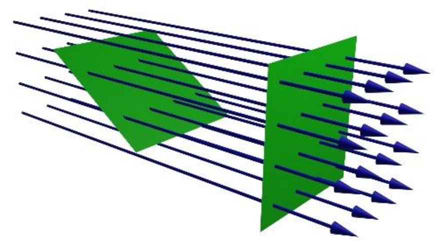
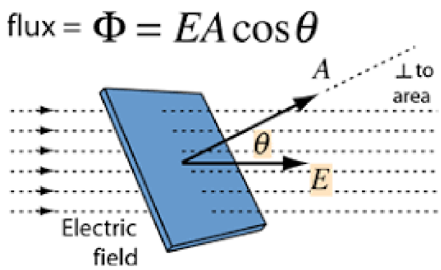
Electric Flux

1. Electric Flux Definition

Electric Flux is the amount of electric field going across a surface.

The electric field lines look like lines of a "fluid". So, you can imagine these lines are flowing (even though nothing is really flowing). The word **FLUX** roughly means **FLOW**.

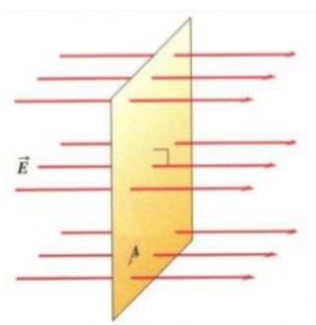




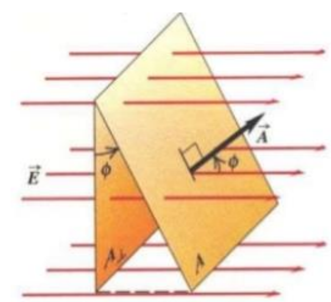
Area Vector $\hat{A}$: the direction is perpendicular with the surface

Definition: $\emptyset_E = \vec{E} \cdot \vec{A} = EA\ Cos\theta$

- ✓ E is Electric Field Strength
- ✓ *A* is the surface area
- ✓ θ is angle between $\hat{E}$ and $\hat{A}$ ($\perp$ with the surface)
- ✓ Unit: Nm^2/C



$\Phi_E = EA$

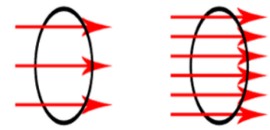


$A_\perp = A\cos\phi$ so $\Phi_E = EA_\perp = EA\cos\phi$

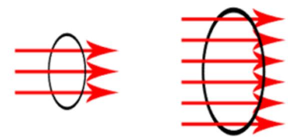
	Electric Flux $\emptyset_E$
Definition	**Electric Flux** is the amount of electric field going across a surface
Magnitude	$\emptyset_E = \vec{E} \cdot \vec{A} = EA_\perp = EA\ Cos\theta$
Vector / Scalar	
Unit	
Base Unit	

2. Electric Flux Characteristic

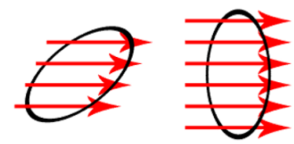
- Flux is proportional to the density of flow:



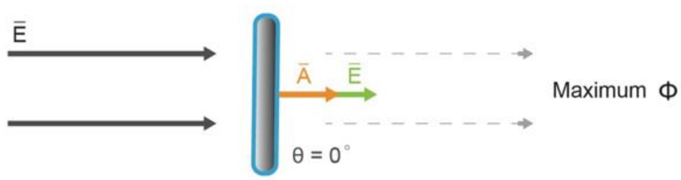
- Flux is proportional to the area within the boundary of flow:



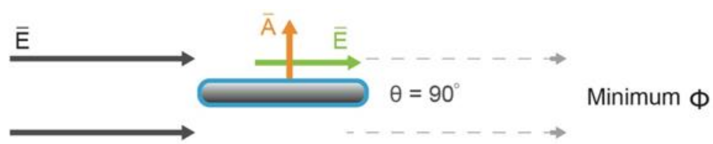
- Flux varies by how the boundary faces the direction of flow:



- When surface is perpendicular with the flow (θ = 0), the flux has the maximal value $\emptyset_{\boldsymbol{E}} = EA$.

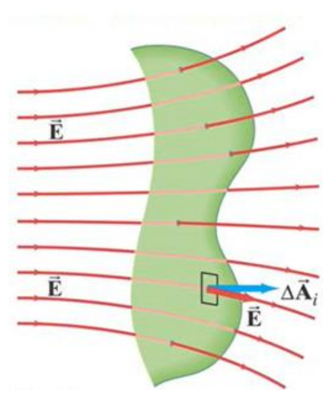


- When surface is parallel with the flow (θ = 90°), the flux has the minimal value $\emptyset_{\boldsymbol{E}} = 0$



3. Electric Flux of un-uniform electric field and not flat surface

- Any area can be divided up into *n* number of small elements of surface area ΔA.
- Each area is sufficiently small to be considered flat and the E-field is uniform over this small area.
- Then :



$$\emptyset_{\boldsymbol{E}} = \overrightarrow{\boldsymbol{E_1}} \cdot \overrightarrow{\boldsymbol{A_1}} + \overrightarrow{\boldsymbol{E_2}} \cdot \overrightarrow{\boldsymbol{A_2}} + \overrightarrow{\boldsymbol{E_3}} \cdot \overrightarrow{\boldsymbol{A_3}} + \ldots + + \overrightarrow{\boldsymbol{E_n}} \cdot \overrightarrow{\boldsymbol{A_n}}$$

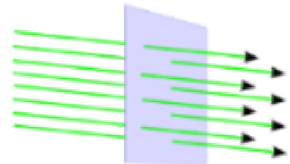
$$= \sum_1^n \overrightarrow{\boldsymbol{E_\iota}} \cdot \overrightarrow{\boldsymbol{A_\iota}}$$

- Integral expression**:**

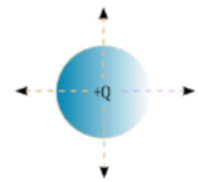
$$\emptyset_{\boldsymbol{E}} = \int \overrightarrow{\boldsymbol{E}} \cdot \overrightarrow{\boldsymbol{dA}}$$

4. Practice

1. A uniform electric field E = 8000 N/C passing through a flat square area A = 10 m^2. Determine the electric flux.

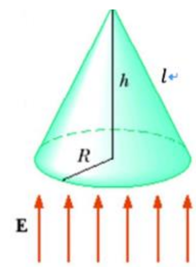


2. A solid ball with 0.5 meters radius has 10 µC electric charge in its center. Determine the electrical flux pass through the solid ball.



<u>Question 3 – 5:</u>

The cone is located in the uniform electric field.



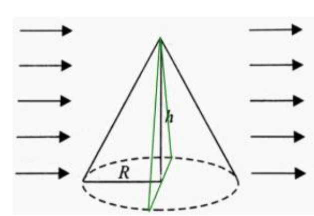
3. What's the electric flux through the corn base area?

4. What's the electric flux through the lateral portion (slanted sides) of the corn?

5. What's the net electric flux through the corn?

6. A cylinder with radius R is located in a uniform electric field E, where the axis of the cylinder is parallel to the electric field. What's the flux through the cylinder surface?

7. A corn lines in a uniform electric field E. What's the electric flux entering the cone?



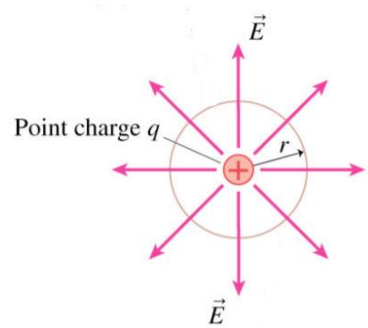
Answer:

1. 8×10^4 Nm^2/C
2. 1.13×10^6 Nm^2/C
3. $\pi R^2 E$
4. $\pi R^2 E$
5. 0
6. 0
7. RHE

Gauss Law

1. Flux due to point charge inside a closed surface

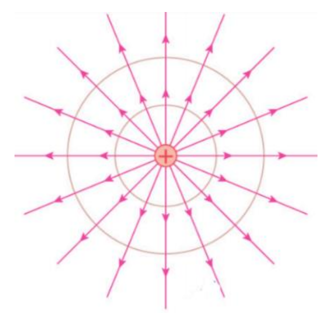
Q1: A point charge q is located the center of a closed sphere with radius r, what's the flux pass through this sphere surface?



- The electric field generated by point q *is* everywhere perpendicular to the surface.
- E has the same magnitude at every point:
  $E = \frac{kq}{r^2} = \frac{q}{4\pi\varepsilon_0 r^2}$
- The flux pass through the sphere is:
  $\emptyset_E = \oint\!\!\oint \vec{E} \cdot \overrightarrow{dA} = EA_{\text{sphere}} = E \times 4\pi R^2$

  Then: $\boldsymbol{\emptyset_E = \frac{q}{\varepsilon_0}}$

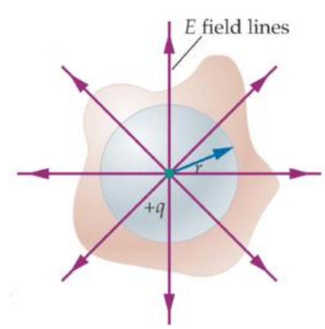
Q2: Another sphere with a larger radius, and a point charge q is located at its center. What's the flux pass through the large sphere surface?



- Every field line pass through the smaller sphere also passes through the large sphere. Herse the flux through the two sphere is the same.

$$\emptyset_E = \frac{q}{\varepsilon_0}$$

Q3: A point charge q is located inside an arbitrary shape, what's the flux pass through the shape surface?



- Every field line pass through the sphere also passes through any arbitrary shape surface. Herse the flux through any closed surface is the same.

$$\emptyset_E = \frac{q}{\varepsilon_0}$$

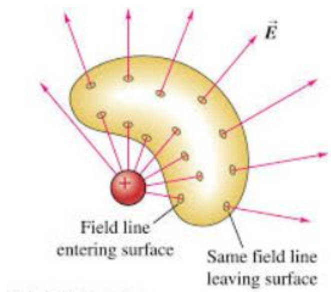
Conclusion:

The flux due to point charge inside a closed surface:

- Independent of radius, size and shape.
- Decided by the inside charge.
- $\emptyset_E = \oint\!\!\oint \vec{E} \cdot \overrightarrow{dA} = \frac{q}{\varepsilon_0}$

2. Flux due to charges outside of a closed surface

A point charge located outside a closed surface. The number of lines entering the surface equals the number leaving the surface.



Conclusion:

The net flux due to charge outside a closed surface is always 0, because the enclosed charge is 0.

3. Gauss Law

The total of the electric flux out of a **closed** surface is equal to the charge **enclosed** divided by the permittivity.

$$\emptyset_E = \oint\!\!\oint \vec{E} \cdot \overrightarrow{dA} = \frac{Q_{enclosed}}{\varepsilon_0}$$

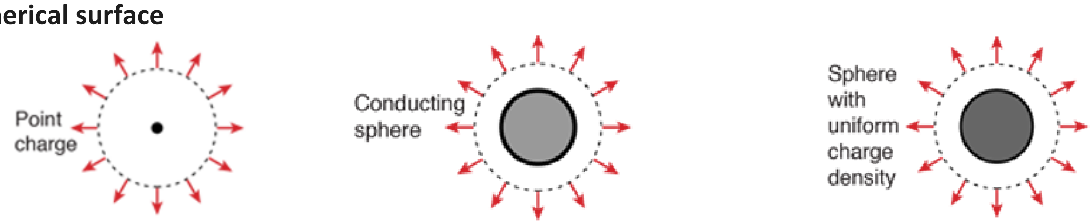
Gauss' law is a powerful tool for the calculation of electric fields when they originate from charge distributions of enough symmetry to apply it.

4. Gaussian surface

A **Gaussian surface** (sometimes abbreviated as G.S.) is a closed surface in three-dimensional space through which the flux of a vector field is calculated.

- Gaussian surfaces are usually carefully chosen to exploit symmetries of a situation to simplify the calculation of the surface integral.
- If the Gaussian surface is chosen such that for every point on the surface the component of the electric field along the normal vector is constant, then the calculation will not require difficult integration.

Spherical surface



A spherical Gaussian surface is used when finding the electric field or the flux produced by any of the following

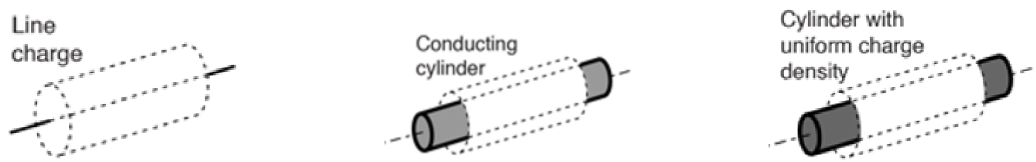
- a point charge
- a uniformly distributed spherical shell of charge
- any other charge distribution with spherical symmetry

The spherical Gaussian surface is chosen so that it is concentric with the charge distribution

The flux out of the spherical surface S is:

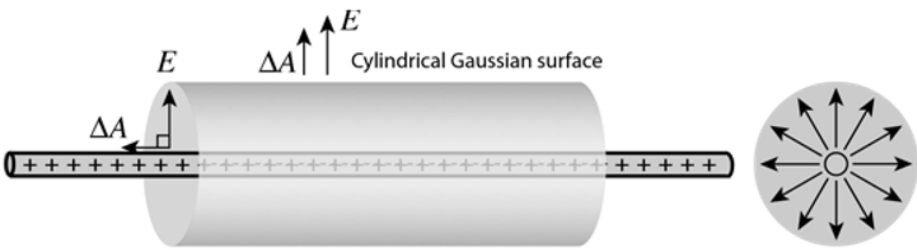
$$\emptyset_E = \oint\!\!\oint \vec{E} \cdot \overrightarrow{dA} = E \times A_{sphere} = E4\pi R^2$$

Cylindrical surface



A cylindrical Gaussian surface is used when finding the electric field or the flux produced by any of the following:

- an infinitely long line of uniform charge
- an infinite plane of uniform charge
- an infinitely long cylinder of uniform charge



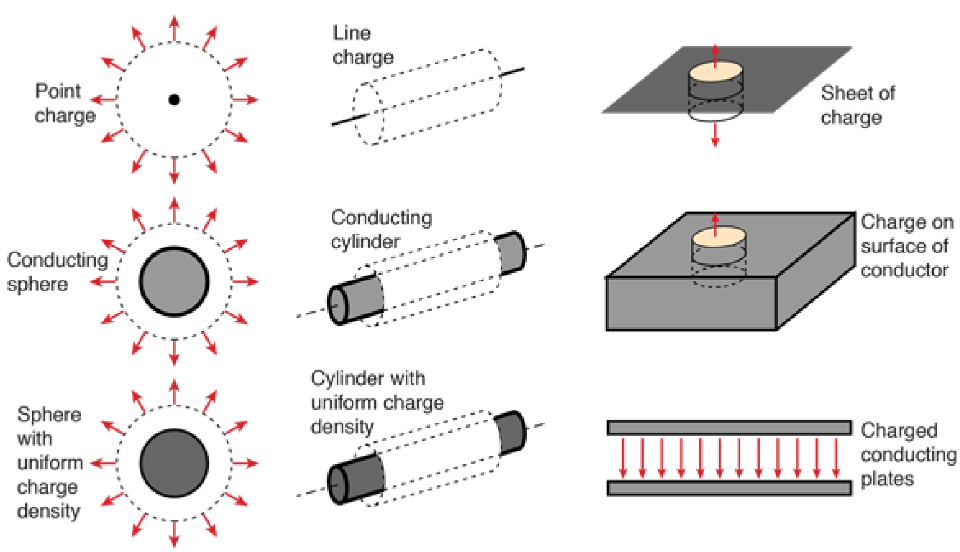
The flux out of the cylinder consists of the three contributions:

$$\emptyset_E = \oiint \vec{E} \cdot \overrightarrow{dA} = \iint_{base} \vec{E} \cdot \overrightarrow{dA} + \iint_{side} \vec{E} \cdot \overrightarrow{dA} + \iint_{base} \vec{E} \cdot \overrightarrow{dA}$$

- Electric field *E* is parallel with the base, so the flux passing the base surfaces is 0;
- Electric field *E* is perpendicular with the side and magnitude is equal every point at side surface, so the flux passing the side surfaces is *E x 2πrL*; (*r* is the radius of base and *h* is the height of cylinder)

So, $\emptyset_E$ = *E2πrL*

Common Gaussian surface



Applications of Gauss' Law

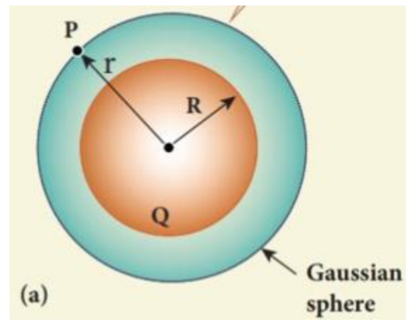
1. Uniformly Charged Sphere

A sphere of radius R has a uniform volume charge Q. Find the electric field at a point P outside the sphere ($r \geq R$) and at a point P inside the sphere ($r < R$).

1) For points outside the sphere, we can draw a large, spherical surface (Gaussian Surface) concentric with the sphere.
   - The Gaussian Surface area $A = 4\pi r^2$
   - The enclosed charge is Q

   $\emptyset_E = \oiint \vec{E} \cdot \overrightarrow{dA} = E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$

   Then, $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$



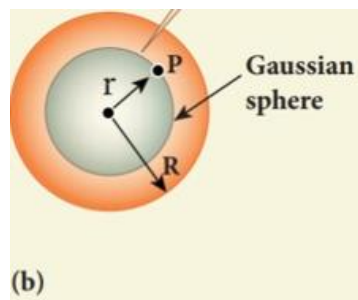
2) For points inside the sphere, we can draw a spherical surface (Gaussian Surface) smaller than sphere.
   - The Gaussian Surface area $A = 4\pi r^2$
   - The enclosed charge is **NOT** Q

   $Q_{enclosed} = \rho V = \frac{Q}{\frac{4}{3}\pi R^3} \times \frac{4}{3}\pi r^3 = \frac{Qr^3}{R^3}$

   ρ is volume charge density (charge per unit volume)

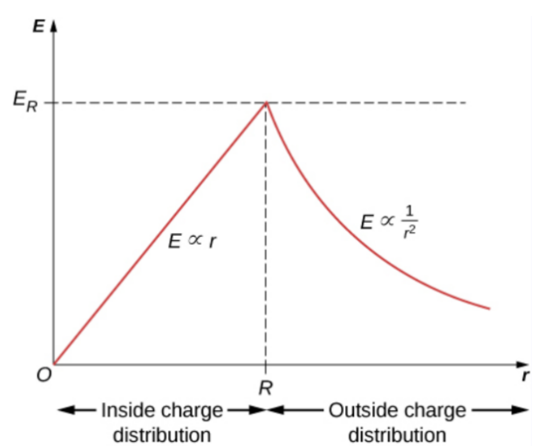
   $\emptyset_E = \oiint \vec{E} \cdot \overrightarrow{dA} = E \cdot 4\pi r^2 = \frac{Q_{enclosed}}{\varepsilon_0}$

   Then, $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^3}r$



Conclusion:

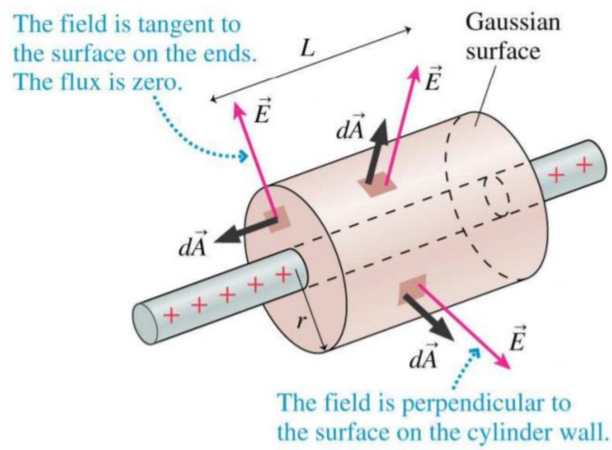
- ➢ The electric field inside the sphere increases linearly with the radius, because the amount of the charge enclosed by the Gaussian surface increases with the volume.
- ➢ The electric field outside of the sphere decreases with the radius, because the amount of the included charge remains the same but the distance increases.
- ➢ The electric field is maximal at the sphere surface.



2. Uniformly Charged Cylinder

An infinite cylinder of radius R has a uniform linear charge density λ (charge per unit length). Find the electric field at a point P outside the cylinder ($r \geq R$).

For points outside the cylinder, we can draw a large, cylindrical surface (Gaussian Surface) with radius r and length L, and concentric with the charged cylinder.



- $\emptyset_E = \oiint \vec{E} \cdot \overrightarrow{dA} = \iint_{side} \vec{E} \cdot \overrightarrow{dA} = E2\pi rL$
- The enclosed charge $Q_{enclosed} = \lambda L$

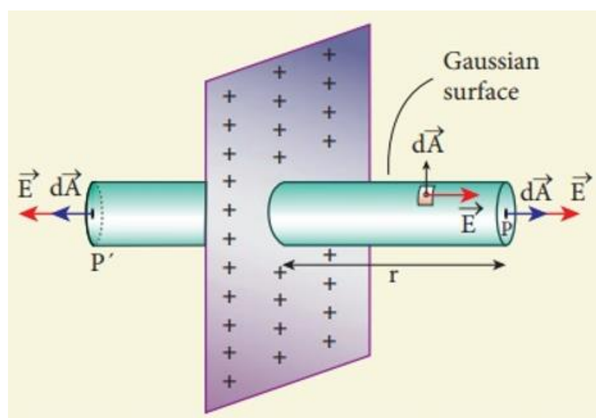
$$\emptyset_E = \oiint \vec{E} \cdot \overrightarrow{dA} = \frac{Q_{enclosed}}{\varepsilon_0}$$

$$E2\pi rL = \frac{\lambda L}{\varepsilon_0} \implies E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

3. Uniformly Charged Sheet

An infinite sheet has a uniform area charge density σ(charge per unit area). Find the electric field.

We can draw a cylindrical surface (Gaussian Surface) with radius r and length L, and perpendicular with the sheet.

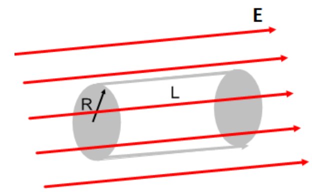


- Electric field E is parallel with the side, so the flux passing the side surfaces is 0;
- Electric field E is perpendicular with the base and magnitude is equal every point at base surface, so the flux passing the base surfaces is: $\emptyset_E = E \times 2A_{base} = E2\pi r^2$
- The enclosed charge $Q_{enclosed} = \sigma A_{base} = \sigma\pi r^2$

$$\emptyset_E = \oiint \vec{E} \cdot \overrightarrow{dA} = \frac{Q_{enclosed}}{\varepsilon_0}$$

$$E2\pi r^2 = \frac{\sigma\pi r^2}{\varepsilon_0} \implies E = \frac{\sigma}{2\varepsilon_0}$$

Practice

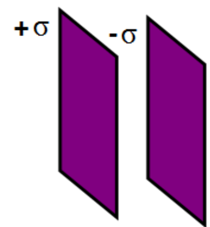


1. A closed surface, in the shape of a cylinder of radius R and Length L, is placed in a region with a constant electric field of magnitude E. The total electric flux through the cylindrical surface is

(A) $E\pi R^2$ (B) zero (C) $E\,2\pi R\,L$ (D) $E\,\pi R^2 L$ (E) $E\,\pi RL$

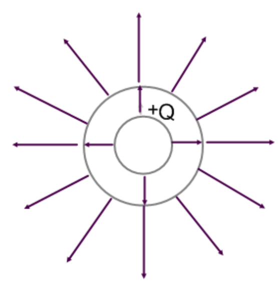
2. Which of the following represents the electric field due to an infinite charged sheet with a uniform charge distribution σ.

(A) zero (B) σ (C) $\sigma/2\varepsilon_0$ (D) $2\sigma/\varepsilon_0$ (E) σ/ε_0



3. Two large parallel sheets charged with equal but opposite charges. What is the electric field in the space between the sheets?

(A) zero (B) σ/ε_0 (C) $2\sigma/\varepsilon_0$ (D) $\sigma/2\varepsilon_0$ (E) σ



4. Two conducting concentric spherical shells are shown right. The electric field due to the charged shells is presented by field lines. What is the charge on the outer shell if the inner shell is charged with a positive charge Q.

(A) -2 Q (B) -Q (C) 0 (D) Q (E) 2 Q

5. Which of the following statements about conductors under electrostatic conditions is true?

(A) Charge placed on the conductor always spreads evenly throughout the entire volume.
(B) Charge placed on the conductor always spreads evenly over the surface.
(C) The charge surface density is always a constant.
(D) The electric field at the surface of a conductor is tangent to the surface.
(E) The electric field at the surface of a conductor is perpendicular to the surface.

6. The net electric flux through a closed surface
   (A) depends on the size of Gaussian surface
   (B) depends on the shape of Gaussian surface
   (C) is zero if only negative charges are enclosed by the Gaussian surface
   (D) is zero if only positive charges are enclosed by the Gaussian surface
   (E) is zero if the net charge enclosed by the Gaussian surface is zero

7. A conducting sphere of radius R carries a charge Q. Another conducting sphere has a radius R/2, but carries the same charge. The spheres are far apart. The ratio of the electric field near the surface of the smaller sphere to the field near the surface of the larger sphere is most nearly

(A) 1/4 (B) 1/2 (C) 1 (D) 2 (E) 4

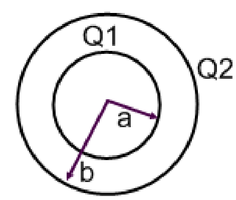
<u>Questions 8-9</u>

Two concentric, spherical conducting shells have radii a and b and charges Q_1 and Q_2, as shown below.

8. In the region where $r < a$, the electric field is proportional to

(A) Q_1/r^2 (B) $(Q_1 + Q_2)/b^2$ (C) zero

(D) $Q_1/a + Q_2/b$ (E) $Q_1/a^2 + Q_2/b^2$



9. In the region $a < r < b$ the electric field is proportional to

(A) Q_1/r^2 (B) $(Q_1 + Q_2)/r^2$ (C) zero

(D) $Q_1/a + Q_2/b$ (E) $(Q_1+Q_2)/r$

10. A solid nonconducting sphere of radius R has a charge Q uniformly distributed throughout its volume. A Gaussian surface of radius r with $r < R$ is used to calculate the magnitude of the electric field E at a distance r from the center of the sphere. Which of the following equations results from a correct application of Gauss's law for this situation?

(A) $E(4\pi R^2) = Q/\varepsilon_o$ (B) $E(4\pi r^2) = Q/\varepsilon_o$ (C) $E(4\pi r^2) = (Q^3r^3)/(\varepsilon_o 4\pi R)$

(D) $E(4\pi r^2) = (Qr^3)/(\varepsilon_o R^3)$ (E) $E(4\pi r^2) = 0$

11. A sphere of radius R has positive charge Q uniformly distributed throughout its volume. Which of the following is the electric field inside the sphere $r < R$?

(A) 0 (B) kQr/R^3 (C) kQ/r^2

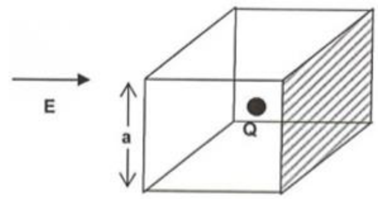
(D) kQ/R^3 (E) kQ/r

12. A sphere of radius R has positive charge Q uniformly distributed throughout its volume. Which of the following is the electric field outside the sphere r > R?

(A) 0 (B) kQr/R3 (C) kQ/r^2

(D) kQ/R^3 (E) kQ/r

13. There's a positive point charge Q inside the cube, and the external uniform electric field E is perpendicular to the shaded area. The side of cube is a. what's the total electric flux through the shaded area?



(A) Q/ε_o (B) 0

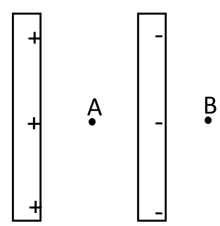
(C) $Q/6\varepsilon_o$ (D) $Q/6\varepsilon_o + Ea^2$

14. Gauss's law provides a convenient way to calculate the electric field outside and near each of the following isolated charged conductors EXCEPT a

(A) large plate (B) sphere (C) cube (D) long, solid rod (E) long, hollow cylinder

Questions 15-16

The diagram right shows 2 oppositely charged parallel plates each with a uniform electric field E.



15. What is the magnitude and direction of the electric field at point A?

(A) E to the left (B) E to the right (C) zero no direction

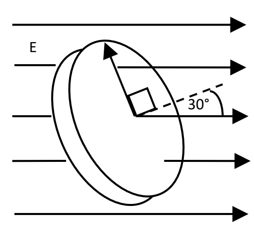
(D) 2E to the right (E) 2E to the left

16. What is the magnitude and direction of the electric field at point B?

(A) E to the left (B) E to the right (C) zero no direction

(D) 2E to the right (E) 2E to the left

17. A uniform electric field E with magnitude 3.0×10^3 N/C passes through a disk of radius 0.2 m 30° above the normal line as shown right. Approximately what is electric flux through the disk?

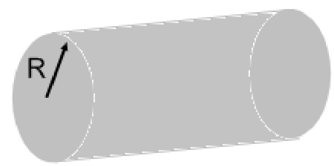


(A) 0 Wb (B) $60\pi\sqrt{3}$ Wb (C) $30\pi\sqrt{3}$ Wb

(D) $20\pi\sqrt{2}$ Nm^2/C (E) $80\pi\sqrt{5}$ Wb

Questions 18-20

The figure right shows a cylindrical distribution of charge Q with radius R and constant charge density ρ. (Assuming L>>R)



18. Which of the following is the electric field inside the cylinder?

(A) $\frac{\rho r}{\varepsilon_0}$ (B) $\frac{\rho r}{2\varepsilon_0}$ (C) zero

(D) $\frac{\rho R}{\varepsilon_0 r}$ (E) $\frac{\rho r}{4\varepsilon_0 R}$

19. Which of the following is the electric field outside the cylinder?

(A) $\frac{\rho R^2}{2\varepsilon_0 r}$ (B) $\frac{\rho r}{\varepsilon_0}$ (C) zero (D) $\frac{\rho r}{2\varepsilon_0}$ (E) $\frac{\rho R}{4\varepsilon_0}$

Answers

1. B
2. C
3. B
4. E
5. E
6. E
7. E
8. C
9. A
10. D
11. B
12. C
13. D
14. C
15. D
16. C
17. B
18. B
19. A